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THE PROPAGATION AND THE WAVE FORM OF THE VAPOR VOLUMETRIC CONCENTRATION IN BOILING, FORCED CONVECTION SYSTEM UNDER OSCILLATORY CONDITIONS

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Abstract—Various aspects and characteristics of the void propagation equation are discussed. This equation predicts the transient response of the volumetric concentration to perturbations of (1) power input, (2) inlet flow, (3) system pressure, (4) thermodynamic nonequilibrium, (5) compressibilities of the vapor and of the liquid and (6) body forces acting on the two-phase mixture. This transient response is predicted both as function of space and function of time.

Solutions of the void propagation equation are derived for the following operating conditions: (1) constant power and inlet flow, (2) oscillatory power input, (3) oscillatory inlet flow and (4) oscillatory power and oscillatory flow.

It is shown that perturbations of the mixture density are propagated through the two-phase mixture by the *velocity of kinematic waves*. Expressions which predict the rate of propagation of these waves and which are appropriate to the operating conditions listed above are presented.

The finite rate of propagation of kinematic waves introduces a "delay time" which characterizes the response of the volumetric concentration to various perturbations. The "delay times", appropriate to the operating conditions enumerated above, are also presented.

The predicted results are compared to available experimental data, satisfactory agreement is shown.

NOMENCLATURE

MLTΘ system of units, with H defined by $H = ML^2/T^2$.

A_c, cross-sectional area $[L^2]$;
C_k, velocity of kinematic wave $[L/T]$;
C_0, distribution perimeter [—];
D, diameter of the duct $[L]$;
E, energy $[H/M]$;
g, acceleration due to gravity $[L/T^2]$;
h, heat-transfer coefficient $[HL^{-2}\,T^{-1}\,\Theta^{-1}]$;
i_f, enthalpy of the liquid $[H/M]$;
i_g, enthalpy of the vapor $[H/M]$;
Δi_{fg}, latent heat of vaporization $[H/M]$;
j, volumetric flux density $[L^3/TL^2]$;
L_b, boiling length $[L]$;
P, pressure $[M/T^2]$;
t, time $[T]$;
ΔT, temperature difference;
v, velocity $[L/T]$;
V_{gj}, drift velocity of the vapor $[L/T]$;
V_{fj}, drift velocity of the liquid $[L/T]$;
$\frac{D_f}{D_t}$, $= \frac{\partial}{\partial t} + v_f \frac{\partial}{\partial z}$, total derivative following liquid particle;
$\frac{D_g}{D_t}$, $= \frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z}$, total derivative following vapor particle;
z, distance in the axial direction $[L]$.

Greek symbols

α, vapor volumetric concentration [—];
Γ, rate of mass formation per unit volume $[M/\Theta L^3]$;
σ, surface tension $[ML/T^2L]$;
ρ, mass density $[M/L^3]$;
$\Delta\rho$, $\rho_f - \rho_g$ $[ML^{-3}]$;

Ω, characteristic reaction frequency $[T^{-1}]$;
τ, evaporation time constant $[T]$;
ζ, heated perimeter $[L]$;
ω_f, frequency of inlet flow oscillation $[T^{-1}]$;
ω_p, frequency of input power oscillation $[T^{-1}]$.

Subscripts
f, liquid;
g, vapor;
m, mixture;
r, relative;
fi liquid inlet.

Dimensionless group
ϵ_p, ϵ_f, power and inlet flow oscillation amplitude;
v'^*, see equation (61);
Γ^*, see equation (30);
Γ_0^*, see equation (41);
α^*, see equation (32);
t^*, see equation (29);
z^*, see equation (38);
ω^*, see equation (50);
C_k^*, see equation (51);
$\langle\ \rangle$, average value across duct cross-section.

1. INTRODUCTION

1.1 *Previous work*

THE ABILITY to predict the transient response of the volumetric concentration in a two-phase system is of considerable importance to present technology. For example, analyses and accurate predictions of the dynamic characteristics of nuclear reactors, space power plants, marine propulsion systems, chemical process apparatus, etc., depend on the correct formulation of the transient response of the volumetric concentration. Consequently, there have been numerous publications concerned with this problem.

The different methods which have been used to analyse the problem are discussed in more detail elsewhere [1, 2]. It was noted there that the problem was formulated either in terms of questionable methods or in terms of an incomplete set of equations describing the conservation laws for the mixture.

With exception of reference [3] all analyses which consider the conservation equations were formulated in terms of (1) the momentum equation for the mixture, (2) the energy equation of the mixture and (3) *one* equation of continuity, i.e. the continuity equation for the mixture. This is most surprising since it is well-known† that for multi-component or multi-phase system the number n, of continuity equations is equal to the number n of the components or of the phases. It is customary to add these n equations in *one* continuity equation for the mixture, and to express the remaining $n-1$, equations as *diffusion equations*. With exception of reference [3], who consider a diffusion equation for the vapor, this was never done in analyses of boiling, two-phase flow systems. As noted above, all analyses dealing with the transient response of a boiling mixture were formulated in terms of only *one* equation of continuity, i.e. the continuity equation for the mixture. No reference was ever made to either the continuity equations of the constituents or to the resulting diffusion equation.

1.2 *Purpose of this article*

The questions of interest in an analysis of the transient behavior of a two-phase flow system are:

(1) How can the change of volumetric concentration α, as it passes through the system be predicted in advance?
(2) When will such a change reach a given point in the system?
(3) As the variation of the volumetric concentration α, moves along the duct will the variation spread out or will it become more concentrated, and how fast?

† See, for example, reference [4].

In analyses of multi-component systems (for example mixture of gases) it has been customary in the past to seek the answer to the three questions above by formulating the problem in terms of Fick's diffusion equation and by determining the diffusion coefficient from experiment. The same approach could be used in analysing the transient response of a forced convection, boiling system were it not for the fact that no data are available in the literature on the diffusion coefficient in such a system.

Instead of seeking a solution in terms of the diffusion equation, it was shown analytically and verified experimentally in reference [5] (by bubbling air through water) that the kinematic wave theory [6, 7] provides a convenient answer to the three questions posed above. The kinematic wave theory was developed by Lighthill and Whitham [6, 7] for analysing flood waves and traffic flow on highways. Indeed, the questions which were raised above and which are of interest to the chemical process industry and to the nuclear reactor technology, are identical with those raised by Lighthill and Whitham in connection with the flow of cars.

The kinematic wave theory was apparently first applied [8] to analyse the transient response of a dispersed two-phase (solid–gas) system. It has been successfully applied to analyse both the transient response and the operating limits of fluidized systems in the *absence* of a change of phase [9–11].† The relation between an analysis formulated in terms of kinematic waves and an analysis formulated in terms of the diffusion equation is given in references [11–13]; it is also summarized in the Appendix A.

A general expression which can be used to predict the transient response of the vapor volumetric concentration in a two-phase system with a *change of phase* has been derived [1, 2]. The resulting void propagation equation, which was formulated and expressed in terms of kinematic waves, gives the response of the vapor volumetric concentration to variation of: (1) power density, (2) pressure, (3) energy storage in the vapor, (4) compressibilities of the liquid and of the vapor, (5) flow rate and (6) gravitational force field, i.e. of the body forces. This general expression for predicting the void response is compared [1, 2] to the analytical results reported previously and, in particular to the results of Kanai *et al.* [14].

It has been shown [15, 16] that both the rate of propagation as well as the wave form of the void disturbance predicted by the void propagation equation were in good agreement with the experimental data reported. In these experiments, performed in a forced convection loop with boiling Refrigerant 22, the oscillatory power input to the fluid was of the form of $[1 + a^2 \sin^2 \omega t + 2a \sin \omega t]$ whereas the vapor void response was determined by means of X-ray attenuation. The same expression for the power input was used in the computer solution of the void propagation equation in order to compare the predicted results with experimental data. Figure 1, which is reproduced from reference [15, 16] shows such a comparison. It can be seen that the oscillatory volumetric concentration $\langle \alpha \rangle$ predicted by the analysis at different times and at different locations along the heated duct are in agreement with the experimental data.

It is of interest to reactor kinetics and to the chemical industry to predict the vapor void response when the oscillatory power input term is of the form of $(1 + \epsilon_p \sin \omega_p t)$. It is of further interest to predict the vapor void response to inlet flow oscillations of the form $(1 + \epsilon_f \sin \omega_f t)$.

It is therefore the purpose here to derive closed form analytical solutions of the void propagation equation which give the response of the vapor volumetric concentration to these two perturbations. In particular we shall obtain solution of the void propagation equation for the following cases: (1) constant power input, (2) oscillatory power input, (3) oscillatory inlet flow and (4) both power input and inlet flow oscillatory. The solutions are presented in dimensionless form so that they can be applied to various systems of practical interest.

† Further references are given in reference [11].

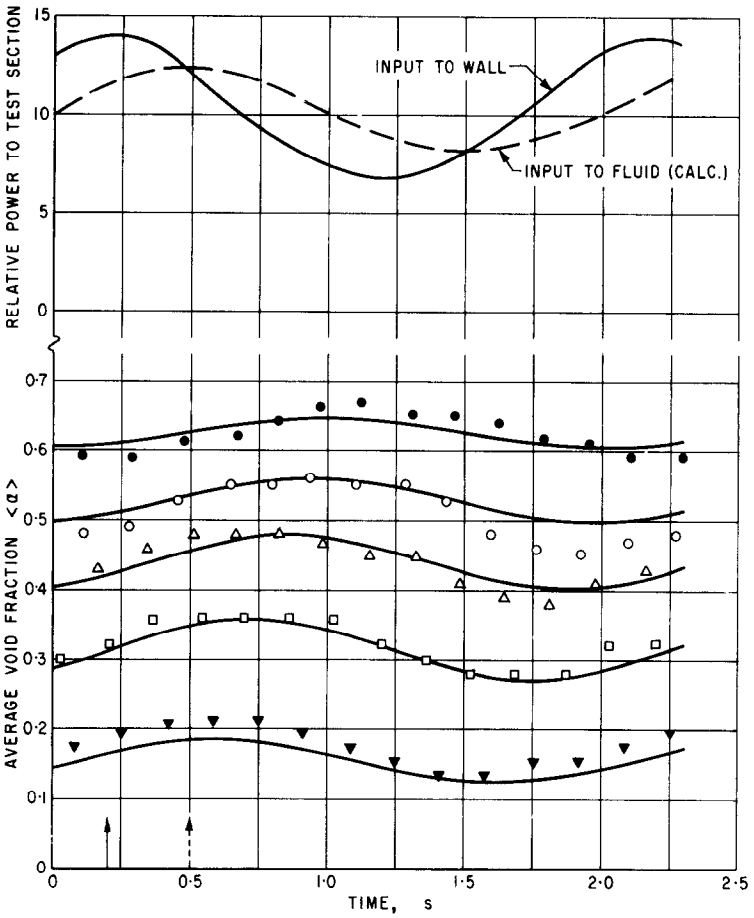

FIG. 1. Comparison of calculated and measured volumetric concentration for oscillating heat input where $\omega t = 0$ at zero time. Reduced pressure = 0·22; average q = 4820 Btu/h ft^2 (see reference [16]). Arrows refer to maximum power input to wall and fluid respectively.

Distance from heated inlet (ft)
▼ 0·52 ○ 2·94
□ 1·13 ● 4·54
△ 2·01

2. FORMULATION OF THE PROBLEM

2.1 *The frame of reference*

In a two-phase flow system the velocities of the two phases are never equal, i.e. there is always a relative motion of one phase with respect to the other. Consequently, a two-phase flow problem must be always formulated in terms of two velocity fields. However, there are several velocity fields which are useful in analysing various aspects of a two-phase flow system.† Depending upon the particular aspect one can select a reference frame and formulate the problem in terms of the velocity fields that are most representative of and appropriate to the solution of that particular problem.

In transient void (or holdup) problems, it is of

† A more detailed discussion is given elsewhere [17].

interest to determine the response of the volumetric vapor concentration to various perturbations such as power, flow, pressure, etc. For this particular two-phase flow problem, it is then advantageous to formulate the analysis [1, 2] in terms of the velocity of the center of volume **j**, and of the drift velocity $\mathbf{V}_{gj}$ and $\mathbf{V}_{fj}$ of the vapor and of the liquid with respect to **j**.

Let $\mathbf{v}_f$ and $\mathbf{v}_g$ be the local point values of the velocities of the liquid and of the vapor and let α be the local point value of the volumetric concentration of the vapor, then we define the volumetric flux densities of the liquid $\mathbf{j}_f$, and the vapor $\mathbf{j}_g$, by

$$\mathbf{j}_f = (1 - \alpha)\mathbf{v}_f \tag{1}$$

$$\mathbf{j}_g = \alpha\mathbf{v}_g \tag{2}$$

and the volumetric flux density of the mixture by:

$$\mathbf{j} = \mathbf{j}_f + \mathbf{j}_g \tag{3}$$

which in view of equations (1) and (2) can be expressed as:

$$\mathbf{j} = (1 - \alpha)\mathbf{v}_f + \alpha\mathbf{v}_g. \tag{4}$$

Two observations are of interest. First, we note that equations (1), (2) and (4) correspond, mathematically, to the definitions of the number velocities in the kinetic theory of gases. Second, we note that equation (4) is an average velocity of the mixture obtained by weighing the respective velocities of the two phases by weight factors $(1 - \alpha)$ and α which are proportional to the two volumes occupied by the liquid and by the vapor phase respectively. Consequently, equation (4) can be interpreted either as the local volumetric flux density of the mixture or as the velocity of the center of volume of the mixture.

The expression for j, which was derived [1, 2] for the case of one-dimensional flow of a two-phase mixture with a change of phase is given by:

$$j = v_{fi} + \int_0^z \left[\frac{\Delta\rho}{\rho_f}\frac{\Gamma_g}{\rho_g} - \frac{(1-\alpha)}{\rho_f}\frac{D_f\rho_f}{Dt} - \frac{\alpha}{\rho_g}\frac{D_g\rho_g}{Dt} \right] dz \tag{5}$$

where v_{fi} is the inlet liquid velocity; ρ_f and ρ_g are the densities of the liquid and of the vapor; Γ_g is the vapor source term which is discussed in the section that follows and total derivatives are defined by:

$$\left.\begin{aligned} \frac{D_f}{Dt} &= \frac{\partial}{\partial t} + v_f\frac{\partial}{\partial z} \\ \frac{D_g}{Dt} &= \frac{\partial}{\partial t} + v_g\frac{\partial}{\partial z} \end{aligned}\right\} \tag{6}$$

Consequently, the last two terms on the right-hand side of equation (5) represent the effect of the compressibilities of the two phases.

It can be seen from equation (5) that the velocity j, of the center of volume, i.e. the volumetric flux density of the mixture at a given point in the system, depends on the inlet velocity and on the *integrated* effect of the vapor generation in the test section decreased by the effects of the compressibilities of the two phases.

Taking now a reference frame which moves with the velocity j, we define the local *drift velocities* with respect to the center of volume of the mixture by:

$$\mathbf{V}_{fj} = \mathbf{v}_f - \mathbf{j} \tag{7}$$

and

$$\mathbf{V}_{gj} = \mathbf{v}_g - \mathbf{j}. \tag{8}$$

It was shown [18–20] that the drift velocity of the vapor depends upon the flow regime of the two-phase mixture. The various expressions for V_{gj}, appropriate to the various flow regime, are given together with a general method for determining the drift velocity. It was also shown that for a number of flow regimes, such as the turbulent bubbly flow, the slug flow etc., the drift velocity V_{gj} does not depend upon the void fraction α. For example, for the churn turbulent bubbly flow, the vapor drift velocity is given by:

$$V_{gj} = 1{\cdot}53\left[\frac{\sigma g\Delta\rho}{\rho_f^2}\right]^{\frac{1}{4}} \tag{9}$$

whereas for the slug flow regime it is given by:

$$V_{gj} = 0{\cdot}35\left[\frac{g\Delta\rho D}{\rho_f}\right]^{\frac{1}{2}} \tag{10}$$

Additional expressions are listed in references [18–20].

2.2 *The vapor source term*

The one-dimensional equation of continuity for the vapor in a two-phase mixture with a change of phase is given by:

$$\frac{\partial \rho_g \alpha}{\partial t} + \frac{\partial[\alpha \rho_g v_g]}{\partial z} = \Gamma_g \tag{11}$$

The vapor source term Γ_g, has the same meaning as the mass source term in the continuity equation for a given species undergoing a chemical reaction. Consequently, in order to specify Γ_g, it is necessary to specify the *constitutive equation* for the process.

The constitutive equation for chemical reactions are given in terms of the reaction rates. For a two-phase mixture, the problem is considerably more complicated, because the constitutive equation will depend not only upon the mode of mass transfer, but also upon the *topology* of the interface, i.e. whether it is spherical, cylindrical, plane, etc. The expression for the vapor source term Γ_g, will *depend* therefore upon the flow regime.

The constitutive equations for a two-phase flow mixture with a change of phase are discussed further elsewhere [17] together with the expressions for Γ_g appropriate to the various flow regimes. For the purpose of this paper we note that the vapor source term Γ_g can be also obtained from the energy equation for the mixture when expressed in term of the convected coordinates [17].

The energy equation for the two-phase mixture is [17]:

$$(1-\alpha)\rho_f \frac{D_f E_f}{Dt} + \alpha\rho_g \frac{D_g E_g}{Dt} + \Gamma_g(E_g - E_f) = h\Delta T\left(\frac{\zeta_h}{A_c}\right) + \frac{\partial P}{\partial t} + \rho_m \frac{\partial \phi}{\partial t} \tag{12}$$

where we have neglected the effects of frictional heating. In equation (12) the convected derivatives are given by equation (6), the density of the mixture ρ_m, is given by:

$$\rho_m = (1-\alpha)\rho_f + \alpha\rho_g \tag{13}$$

whereas the energies E_f and E_g are given by:

$$\left.\begin{aligned} E_f &= i_f + \frac{v_f^2}{2} + \phi \\ E_g &= i_g + \frac{v_g^2}{2} + \phi \end{aligned}\right\} \tag{14}$$

where i_f and i_g are the enthalpies of the liquid and of the vapor.

The term ϕ in equations (12) and (14) is the potential energy. For most terrestrial systems of practical interest body forces other than gravity are unimportant. For these systems then the potential energy is time independent and it is given by:

$$\phi = gz. \tag{15}$$

However, when the body forces are functions of time, as may be the case for *marine* and *space* systems under certain operating conditions, then the potential energy is a function of time whence

$$\frac{\partial \phi}{\partial t} \neq 0. \tag{16}$$

The significance of the various terms, which appear in equation (12), are as follows. The first two terms on the left-hand side account for the lack of thermodynamic equilibrium (i.e. for the subcooling or superheating) in the liquid and in the vapor phase. The third term represents the energy required to generate a given mass of vapor per unit time per unit volume. The first term on the right-hand side represents the power input per unit volume of the mixture. The second term accounts for the effects of system pressure variations on the energy content; whereas the last term accounts for the time dependent body force.

It can be seen from equation (12) that when the terms on the right-hand side are given then the vapor source term can be determined *if* it is assumed that both the liquid and the vapor are in thermodynamic equilibrium.† However, if thermodynamic equilibrium is not attained, then information on the constitutive equation, appropriate to the particular flow regime, is *required.*

2.3 *The void propagation equation*

The void propagation equation for a two-phase flow system with a change of phase which was derived in references [1, 2] is given by:

$$\frac{\partial \alpha}{\partial t} + C_k \frac{\partial \alpha}{\partial z} = \Omega \tag{17}$$

where C_k is the velocity of kinematic waves and Ω is the characteristic reaction frequency. The relation between this equation and the standard formulation in terms of the diffusion equation is discussed in more detail elsewhere [12, 13], it is also summarized in Appendix A. Before proceeding further, it is advantageous to discuss the significance of the various terms in equation (17).

The void propagation equation shows that changes in the volumetric concentration α, are transmitted through the system by the velocity of kinematic waves C_k. This velocity can be expressed as:

$$C_k = j + V_{gj} + \alpha \frac{\partial V_{gj}}{\partial \alpha} \tag{18}$$

or, in view of equation (8), as:

$$C_k = v_g + \alpha \frac{\partial V_{gj}}{\partial \alpha} \tag{19}$$

† Note that the statement *thermodynamic equilibrium* is equivalent to an assumption of a *constitutive equation for evaporation*, i.e. of an equation describing a *particular* process of vapor formation. In this particular case it corresponds to a process where the energy is transferred from the heating surface to the vapor without *any* time delay and energy storage in either the liquid or the vapor [see equation (12) for the case when i_f and i_g are constant].

Three important observations can be made with respect to equations (18) and (19).

(1) Since the vapor drift velocity V_{gj} depends upon the flow regime, equations (17) and (18) show that the transient void response will depend also upon the flow regime. This conclusion has been already verified in the experiments reported in reference [5].

(2) Since changes of the volumetric concentration α, are propagated with the velocity of kinematic waves C_k, equations (17) and (19) show that changes of α will propagate *backwards* or *forwards* with respect to the velocity of the vapor, i.e. $C_k < v_g$ or $C_k > v_g$, depending on whether the vapor drift velocity V_{gj} decreases ($\partial V_{gj}/\partial \alpha < 0$) or increases ($\partial V_{gj}/\partial \alpha > 0$) with increasing α. When V_{gj} does not depend upon α then $C_k = v_g$, and the void perturbations propagate with the local vapor velocity.

(3) Since the velocity of kinematic waves C_k, depends upon the volumetric flux density of the mixture, equations (5) and (18) show that the kinematic wave velocity depends upon the inlet conditions and upon the *integrated* effect of the vapor generation in the test section decreased by the effects of the compressibilities of the two phases.

In view of the foregoing it can be seen that, at a given location, the velocity of the kinematic waves takes into account (1) the effect of flow regime, (2) the entrance effect and (3) the effect of the past history of the mixture as it flows through the heated duct.

The characteristic reaction frequency Ω in equation (12), is given by [1, 2].

$$\Omega = \frac{\rho_m}{\rho_f}\frac{\Gamma_g}{\rho_g} + \alpha(1-\alpha)\left[\frac{1}{\rho_f}\frac{D_f\rho_f}{Dt} - \frac{1}{\rho_g}\frac{D_g\rho_g}{Dt}\right]. \tag{20}$$

It can be seen from equation (20) that the characteristic reaction frequency takes into account the *local* effects of the vapor generation and of the compressibilities of the two phases. This is in contrast to the velocity of kinematic waves which,

as noted above, depends upon the *integrated* effect of these three terms.

Substituting equations (20), (18) and (5) into equation (17), the void propagation equation for a two-phase mixture with a change of phase becomes:

$$\frac{\partial\alpha}{\partial} + \left\{ v_{fi} + V_{gj} + \alpha\frac{\partial V_{gj}}{\partial\alpha} + \int_0^z \left[\frac{\Delta\rho}{\rho_f}\frac{\Gamma_g}{\rho_g} - \frac{(1-\alpha)}{\rho_f}\frac{D_t\rho_t}{Dt} - \frac{\alpha}{\rho_g}\frac{D_g\rho_g}{Dt} \right] \mathrm{d}z \right\} \frac{\partial\alpha}{\partial z} = \frac{\rho_m}{\rho_f}\frac{\Gamma_g}{\rho_g} + \alpha(1-\alpha)\left[\frac{1}{\rho_f}\frac{D_f\rho_f}{Dt} - \frac{1}{\rho_g}\frac{D_g\rho_g}{Dt} \right] \quad (21)$$

with the vapor source term Γ_g given by equation (12) thus

$$\Gamma_g = \frac{1}{E_g - E_f}\left[h\Delta T\left(\frac{\zeta_h}{A_c}\right) + \frac{\partial P}{\partial t} + \rho_m\frac{\partial\phi}{\partial t} - (1-\alpha)\rho_f\frac{D_fE_f}{Dt} - \alpha\rho_g\frac{D_gE_g}{Dt} \right] \quad (22)$$

It can be seen from equations (21) and (22) that the void propagation equation predicts the void response to perturbations of: (1) flow rate, (2) power input, (3) system pressure, (4) thermodynamic nonequilibrium, (5) compressibilities of the two phases and (6) of the body forces.

The volumetric concentration in equation (21) is the value of α at a given point in the system. In practice one is interested in the value of α averaged over the cross-sectional area of the duct. The difference between these two values of α is caused by the nonuniform flow and concentration profiles.

In order to express equation (21) in terms of the average volumetric concentration, we follow the method presented previously [18–20]. We define the value of a quantity F averaged over the cross-sectional area by:

$$\langle F\rangle = \frac{1}{A}\int_A F\,\mathrm{d}A \quad (23)$$

and the weighted mean value of F by

$$\bar{F} = \frac{1}{\langle\alpha\rangle}\frac{1}{A}\int_A \alpha F\,\mathrm{d}A \quad (24)$$

Thus, in view of equation (24), we define the weighted mean drift velocity of the vapor by:

$$\bar{V}_{gj} = \frac{\langle\alpha V_{gj}\rangle}{\langle\alpha\rangle} \quad (25)$$

In most problems of practical interest the compressibilities of the liquid and of the vapor can be neglected. Similarly, to a good approximation, the densities of the two phases remain constant in a cross-sectional area of the duct, i.e. they do not depend upon the radius of the duct. Consequently, it is permissible for the purpose of this paper, to use these two simplifying assumptions when evaluating the average value of α.

By means of equations (23) to (25) and following the presentation [1, 2] the void propagation equation can be expressed in terms of the average volumetric concentration $\langle\alpha\rangle$, thus

$$\frac{\partial\langle\alpha\rangle}{\partial t} + \left[C_0 v_{fi} + \bar{V}_{gj} + \langle\alpha\rangle\frac{\partial\bar{V}_{gj}}{\partial\langle\alpha\rangle} + \frac{C_0\Delta\rho}{\rho_f}\int_0^z \frac{\langle\Gamma_g\rangle}{\rho_g}\,\mathrm{d}z \right]\frac{\partial\langle\alpha\rangle}{\partial z} = \left[1 - \frac{C_0\Delta\rho}{\rho_f}\langle\alpha\rangle \right]\frac{\langle\Gamma_g\rangle}{\rho_g} \quad (26)$$

where the distribution parameter C_0 is defined by

$$C_0 = \frac{\langle\alpha j\rangle}{\langle\alpha\rangle\langle j\rangle} \quad (27)$$

It takes into account the effects of the nonuniform flow and concentration profiles [18–21].

It has been shown [18–20] that in vertical upward flow through a circular duct, when the volumetric concentration is highest at the center of the duct, the distribution parameter C_0, can vary between 1·0 and 1·5. However, when the concentration is highest next to the duct walls

(as may be the case in subcooled boiling) the distribution parameter can have a value *smaller* than unity [18]. It was also shown that, for a number of flow regimes, with established profiles of the flow and of the concentration both C_0 and the weighted mean drift velocity $\bar{V}_{gj}$, of the vapor remain essentially constant but that both change with a change of flow regime, i.e. with a *redistribution* of the flow and concentration. The values of C_0 and of $\bar{V}_{gj}$ appropriate to the regimes as well as the method for determining them were given [18–20].

In what follows we shall solve equation (26), however, in order to generalize the results we shall first express equation (26) in dimensionless form.

2.4 *The dimensionless form of the void propagation equation and the general form of the solution*

In order to render the void propagation equation dimensionless we define the dimensionless length by:

$$z^* = \frac{z}{L_b} \tag{28}$$

where L_b is the length of the duct along which the process of evaporation takes place. In what follows, we shall consider those flow regimes for which the vapor drift velocity does not depend upon time; we define then the dimensionless time by:

$$t^* = \frac{C_0 v_{fi} + \bar{V}_{gj}}{L_b} t \tag{29}$$

the dimensionless vapor source term by:

$$\Gamma^* = C_0 \frac{\Delta\rho}{\rho_f} \frac{L_b}{C_0 v_{fi} + \bar{V}_{gi}} \frac{\langle \Gamma_g \rangle}{\rho_g} \tag{30}$$

and the dimensionless velocity of kinematic waves by:

$$C_k^* = 1 + \int_0^z \Gamma^* \, dz^*. \tag{31}$$

The resulting equation can be simplified if we define the volumetric concentration by:

$$\alpha^* = C_0 \frac{\Delta\rho}{\rho_f} \langle \alpha \rangle. \tag{32}$$

Substituting equations (28) to (32) into equation (26), the dimensionless form of the void propagation equation becomes:

$$\frac{\partial \alpha^*}{\partial t^*} + C_k^* \frac{\partial \alpha^*}{\partial z^*} = (1 - \alpha^*)\Gamma^* \tag{33}$$

When the vapor source term Γ^* does not depend upon the volumetric concentration, then equation (33) is a first-order linear partial differential equation whose solution can be obtained by means of characteristics [22, 23]. The general solution of equation (33) is of the form:

$$u_2 = f(u_1) \tag{34}$$

where

$$u_1[\alpha^*, t^*, z^*] = C_1 \quad \text{and} \quad u_2(\alpha^*, t^*, z^*) = C_2 \tag{35}$$

are solutions of any two independent differential equations which imply the relationships

$$dt^* = \frac{dz^*}{C_k^*} = \frac{d\alpha^*}{(1 - \alpha^*)\Gamma^*} \tag{36}$$

For example, by taking alternately the first and the second equation, the first and the third equation we obtain

$$\frac{dz^*}{dt^*} = C_k^* \tag{37}$$

and

$$\frac{d\alpha^*}{dt^*} = (1 - \alpha^*)\Gamma^* \tag{38}$$

which, for the initial and boundary conditions given by: $z^* = 0, \alpha^* = 0, t^* = t_0^*$, yield the following solutions:

$$t^* - t_0^* = \int_0^{z^*} \frac{dz^*}{C_k^*} \tag{39}$$

and

$$\alpha^* = 1 - \exp\left[-\int_{t_0^*}^{t^*} \Gamma^* \, dt^*\right]. \tag{40}$$

We shall here consider systems in thermodynamic equilibrium, we shall neglect therefore the effects of superheat or of subcooling.† Since we are interested in the effects of power and flow oscillations on the volumetric concentration we shall neglect also the effects of pressure variations and of the time dependent body forces.

In what follows, we shall obtain solutions of the void propagation equation for the following cases: (1) constant power input, (2) oscillatory power input, (3) oscillatory inlet flow and (4) both power input and inlet flow oscillating.

3. CONSTANT POWER INPUT AND INLET FLOW

Neglecting in equation (22) the effects of pressure variations, of variable body forces and of thermodynamic nonequilibrium, and substituting the resulting expression in equation (30), we obtain for the dimensionless vapor source

$$\Gamma_0^* = C_0 \frac{\Delta\rho}{\rho_f} \frac{q}{\rho_g \Delta i_{fg}(C_0 v_{fi} + \bar{V}_{gj})} \left(\frac{\zeta_h L_b}{A_c}\right) \tag{41}$$

with $h\Delta T = q$, where q is the power input to the liquid.

The dimensionless velocity of kinematic waves, given by equation (31), then reduces to

$$C_k^* = 1 + \Gamma^* z^* \tag{42}$$

Substituting this expression in equation (36), the volumetric concentration α^*, as function of distance, is obtained from the second and third term in equation (36), thus

$$\alpha^* = \frac{\Gamma_0^* z^*}{1 + \Gamma_0^* z^*} \tag{43}$$

Introducing in equation (43) the expressions for z^*, α^* and Γ^*, given by equations (28), (32) and (41) respectively we obtain the equation which predicts the vapor volumetric concentration in a uniformly heated duct when the inlet liquid is at saturation temperature, i.e. in absence of subcooling, thus:

$$\langle\alpha\rangle = \frac{(q/\rho_g \Delta i_{fg})(\zeta_h/A_c)z}{C_0 v_{fi} + \bar{V}_{gj} + C_0(\Delta\rho/\rho_f)(q/\rho_g \Delta i_{fg})(\zeta_h/A_c)z} \tag{44}$$

which is of the same form as the expression derived previously [18–20] using a different method.

Substituting equation (42) in (39) and equation (41) in (40), we obtain respectively:

$$C_k^* = \exp\left[\Gamma_0^*(t^* - t_0^*)\right] \tag{45}$$

and

$$\alpha^* = 1 - \exp\left[-\Gamma_0^*(t^* - t_0^*)\right]. \tag{46}$$

The latter equation can be expressed in terms of dimensional quantities; thus for equations (46), (41), (32) and (29), we obtain:

$$C_0 \frac{\Delta\rho}{\rho_f}\langle\alpha\rangle = 1 - \exp\left[-C_0 \frac{\Delta\rho}{\rho_f} \frac{q}{\rho_g \Delta i_{fg}} \left(\frac{\zeta_h}{A_c}\right)(t - t_0)\right]. \tag{47}$$

Both equations (46) and (47) show that the vapor volumetric fraction is an exponential function of time which is a characteristic of chemical reactions. For this problem the reaction frequency is given by

$$\Omega = C_0 \frac{\Delta\rho}{\rho_f} \frac{q}{\rho_g \Delta i_{fg}} \frac{\zeta_h}{A_c}. \tag{48}$$

Figure 2 shows the values predicted by equation (44) together with the experimental data of reference [19] for Refrigerant-22 in forced flow through a circular duct. Figure 3 shows the comparison with the experimental data of reference [24] for water at 400 psi in forced flow through a rectangular duct. In preparing these figures the boiling length L_b was taken equal to the length of the duct. This equality results from the assumption of thermodynamic equilibrium which implies that the inlet liquid is at saturation temperature and that boiling starts at the

† We note that in systems where the inlet temperature is below saturation, the degree of subcooling has a considerable effect on the transient response of the void fraction. The result of an investigation concerned with this aspect of the problem will be reported seaparately.

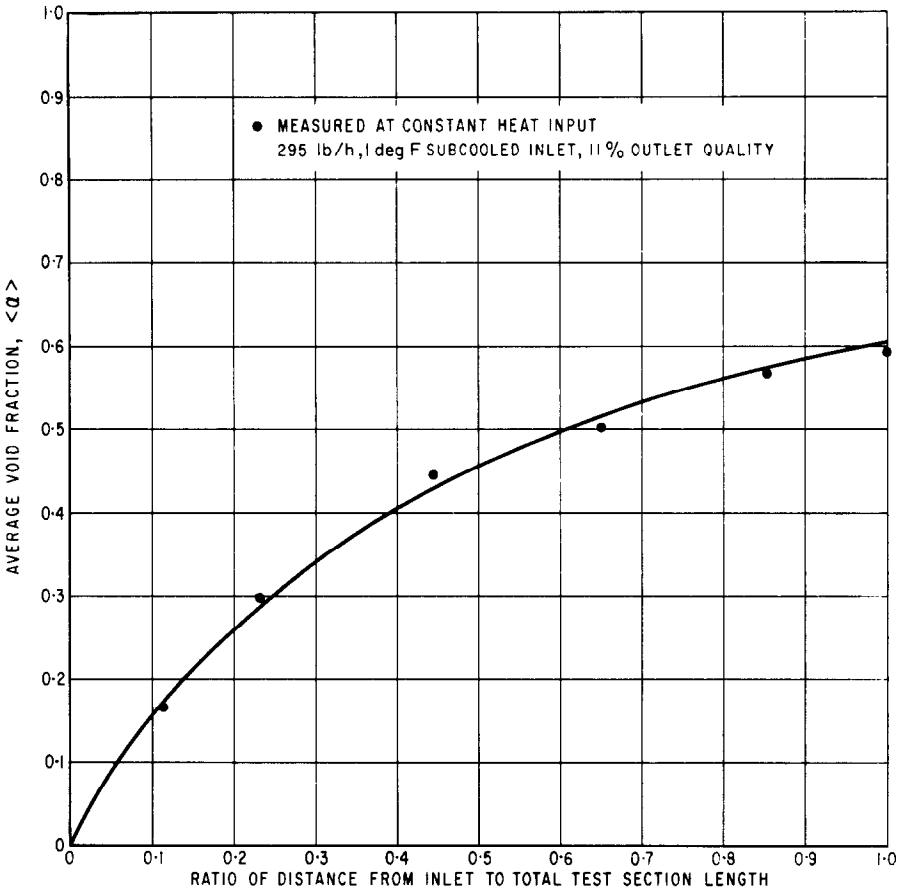

FIG. 2. Comparison of the void fraction for constant power input predicted by equation (44) with experimental data in the churn-turbulent regime for Refrigerant-22 in a round tube [16].

entrance of the heated duct. This was indeed the case for the data shown in Fig. 2. For the data shown in Fig. 3 (run No. 7 in reference [24]) there was a slight subcooling of 1·5 deg F so that the bulk liquid reached saturation temperature at $z^* = 0{\cdot}045$ instead at $z^* = 0$ as assumed in the computations.† We note that had we corrected the quality to take into account this subcooling, the predicted curve would have shifted to the right resulting in a slightly better agreement with the experimental data. Nevertheless, it appears from Figs. 2 and 3 that the predicted results are in satisfactory agreement with the experimental data. Additional comparisons are shown elsewhere [19, 20].

The relations between the expressions that have been derived in this paper and those derived previously [25–28] for the case of homogeneous flow, i.e. for the case when both phases move with the *same* velocity, are given in Appendix B.

4. OSCILLATORY POWER INPUT

Let the oscillatory power input to the liquid be given by:

$$q(t) = q_0(1 + \epsilon_p \sin \omega_p t) \qquad (49)$$

† The other data reported in reference [24] had a higher subcooling, consequently, they are not used here for comparison. They are shown in reference [20] together with an analysis of the void fraction in subcooled boiling.

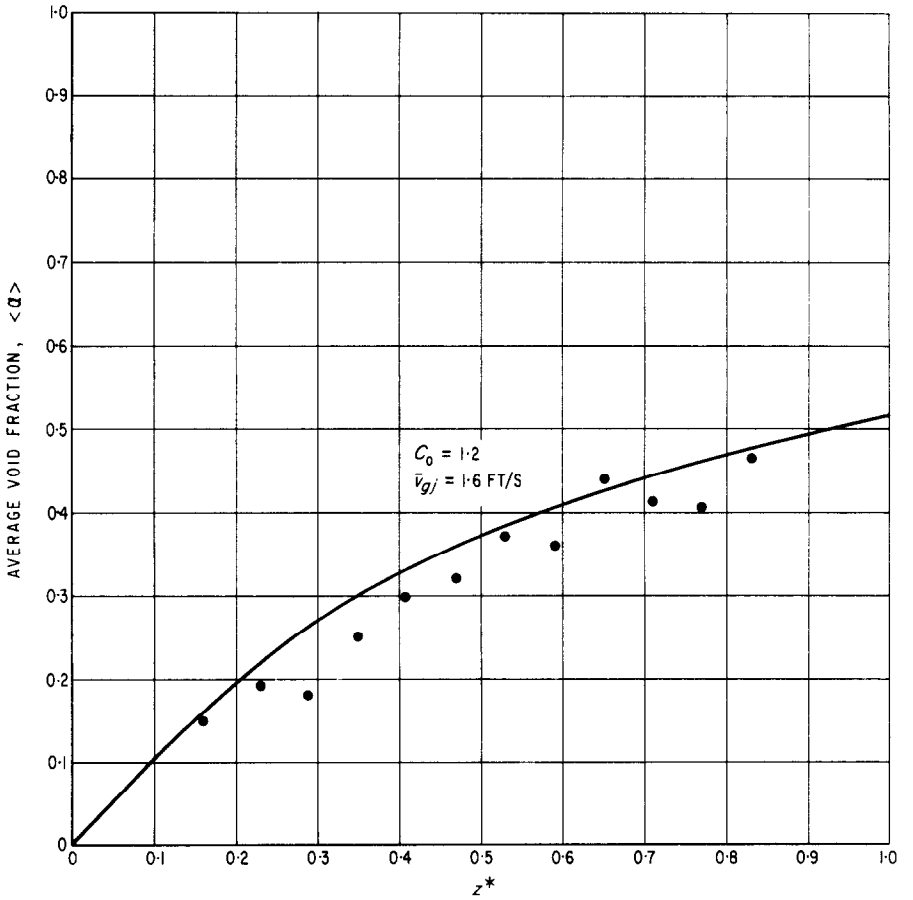

FIG. 3. Comparison of the void fraction for constant power input predicted by equation (44) with experimental data of reference [24] for water in a rectangular channel (see Section 3).

and let the dimensionless frequency of the power oscillation be defined by:

$$\omega_p^* = \frac{\omega_p L_b}{C_0 v_{fi} + \bar{V}_{gj}} \tag{50}$$

Then the dimensionless velocity of kinematic waves, given by equation (31), becomes:

$$C_k^* = 1 + \Gamma_0^*(1 + \epsilon_p \sin \omega_p^* t^*) z^* \tag{51}$$

and the dimensionless vapor source term becomes:

$$\Gamma^* = \Gamma_0^*(1 + \epsilon_p \sin \omega_p^* t^*) \tag{52}$$

where Γ_0^* is given by equation (41).

Substituting equation (52) in (38), i.e. in (40), results in:

$$\alpha^* = 1 - \exp\left[-\Gamma_0^*(t^* - t_0^*) + \frac{\Gamma_0^* \epsilon_p}{\omega_p}(\cos \omega_p^* t^* - \cos \omega^* t_0^*)\right] \tag{53}$$

Substituting equation (51) in (37) gives:

$$\frac{dz^*}{dt^*} - \Gamma_0^*(1 + \epsilon_p \sin \omega_p^* t^*) z^* = 1 \tag{54}$$

whose solution is given by:

$$z^* \exp\left[-\Gamma_0^* t^* + \frac{\Gamma_0^* \epsilon_p}{\omega_p^*} \cos \omega_p^* t^*\right] - \int \exp\left[-\Gamma_0^* t^*\right.$$

$$+ \frac{\Gamma_0^* \epsilon_p}{\omega_p^*} \cos \omega_p^* t^* \Bigg] \mathrm{d}t^* = \text{constant} \quad (55)$$

The integral in equation (55) can be evaluated by successive approximations. However, the advantage of having an analytical solution is then lost because of the complexity of the resulting general solution.

It is advantageous, therefore, to obtain an approximate but simple, closed form solution which, as it will be seen in what follows, is in satisfactory agreement with the exact solution obtained by means of a computer.

The simplification is obtained by letting $\epsilon_p = 0$ in equation (54). This implies that the effect of oscillations is neglected when evaluating the velocity of the kinematic waves given by equation (51). We note that the effect of oscillations is not neglected in the source term. With this assumption C_k reduces to equation (42), and equation (37) yields (45), thus:

$$\Gamma_0^*(t^* - t_0^*) = \ln (1 + \Gamma_0^* z^*) \quad (56)$$

This defines the delay time t_0^*, thus:

$$t_0^* = t^* - \frac{1}{\Gamma_0^*} \ln (1 + \Gamma_0^* z^*) \quad (57)$$

Substituting equation (56) in (53), we obtained α^* as a function of z^* and t^*, thus:

$$\alpha^* = 1 - \frac{\exp \left[(\Gamma_0^* \epsilon_p / \omega_p^*)(\cos \omega_p^* t^* - \cos \omega_p^* t_0^*)\right]}{1 + \Gamma_0^* z^*} \quad (58)$$

In view of equation (32), it can be seen that equation (58) predicts the average volumetric concentration $\langle \alpha \rangle$, at various positions in the heated duct and at various times a function of the oscillatory power input. We note also if we set $\epsilon_p = 0$ in equation (58), we obtain again the solution for case of constant power input given by equation (43).

The values predicted by equation (58) are plotted in Figs. 4–7. The values of the dimensionless frequency ω_p^*, of the dimensionless vapor source Γ_0^* and of the dimensionless power amplitude which have been used in preparing these figures cover a wide range of operating conditions.

Since equation (58) is subject to the assumption of neglecting the effect of oscillations on the velocity of kinematic waves it is of interest to compare the values predicted by equation (58) with the exact solution which takes into account the effect of oscillation on C_k. For this purpose equation (33) was solved on a computer using for C_k^* and Γ^* the expressions given by equations (51) and (52) respectively. The resulting solutions are plotted as full lines in Figs. 4–7. It can be seen from these figures that the values predicted by the approximate but simple, closed form solution, i.e. by equation (58) are in relatively satisfactory agreement with the exact, computer solution. This is especially evident at high values of ω_p^* irrespective of the magnitude of Γ_0^* (see Figs. 4 and 5). However, at a low value of ω_p^* and at a high value of Γ_0^* the amplitude predicted by equation (58) overestimates that predicted by the computer (see Fig. 7).

Figures 8 and 9 show a comparison of the predicted results with the experimental data reported in reference [24] (run No. 7) for oscillatory power input to water at 400 psi. The full lines are the computer solution whereas the dashed curves are the values predicted by equation (58). As in the case of constant power (c.f. Fig. 3) the predicted values of $\langle \alpha \rangle$ are higher than the experimental data for low values of z^*. This is the consequence of inlet subcooling. For higher values of z^*, when the effect of inlet subcooling is negligible, these figures show good agreement between predicted and experimental results. As noted previously an analysis of the void response is subcooled boiling will be reported separately.

It appears from the foregoing that, when the subcooling is negligible either the exact computer solution or equation (58) can be used to predict the transient response of the volumetric concentration to oscillations of the power input to the fluid. This transient response is given both as a function of time and a function of space. This makes the void propagation equa-

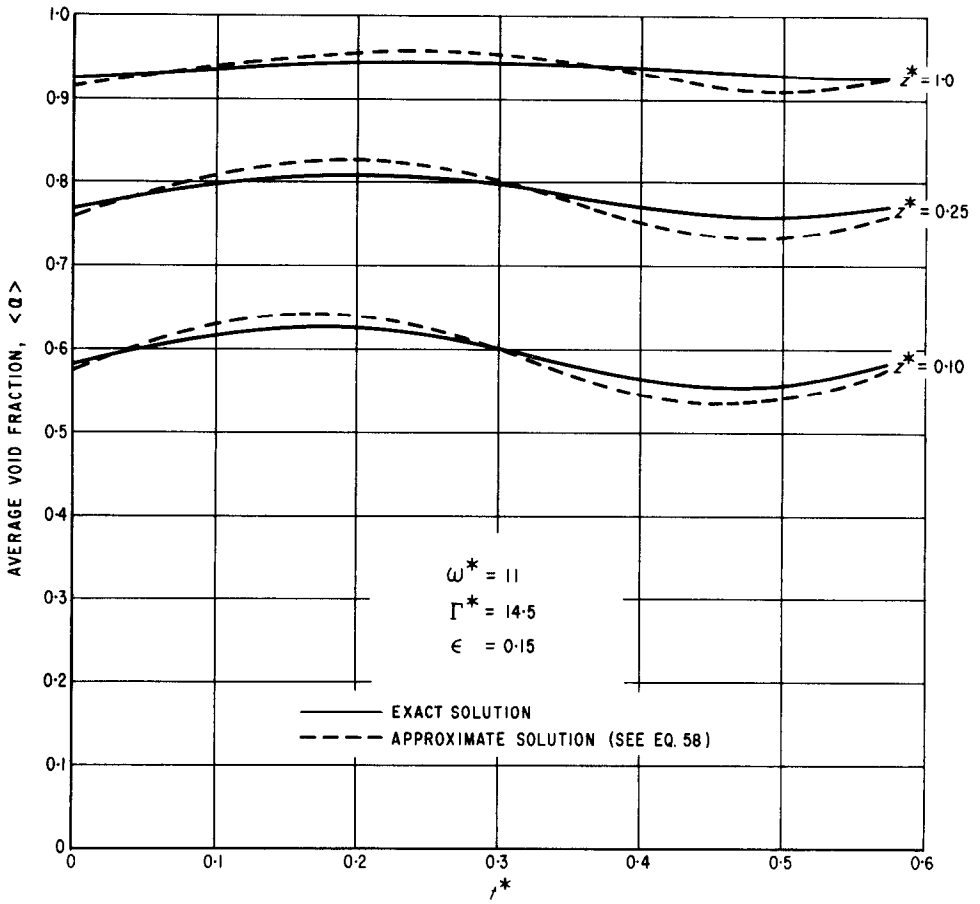

FIG. 4. Comparison of the oscillatory void fraction predicted by equation (58), with the computer solution of equation (33).

tion particularly suitable for analyses of transients in fast, power reactors which use liquid metal for coolant.

5. OSCILLATORY INLET FLOW

We consider now the void response to flow oscillation when the power input is kept constant.

Let the inlet flow oscillation be described by:

$$v_{fi}(t) = v_{fi}(1 + \epsilon_f \sin \omega_f t). \tag{59}$$

We define the dimensionless frequency of the flow oscillation by:

$$\omega_f^* = \frac{\omega_f L_b}{C_0 v_{fi} + \bar{V}_{gj}} \tag{60}$$

and the dimensionless amplitude of the flow oscillation by:

$$v'^* = \frac{C_0 v_{fi} \epsilon_p}{C_0 v_{fi} + \bar{V}_{gj}} \tag{61}$$

Using these dimensionless groups together with those defined in Section 2.3, we obtain from equation (21) the dimensionless velocity of kinematic waves, thus:

$$C_k^* = 1 + v'^* \sin \omega_f^* t^* + \int_0^{z^*} \Gamma^* \, dz^* \tag{62}$$

which for a constant power input reduces to:

$$C_k^* = 1 + v'^* \sin \omega_f^* t^* + \Gamma_0^* z \tag{63}$$

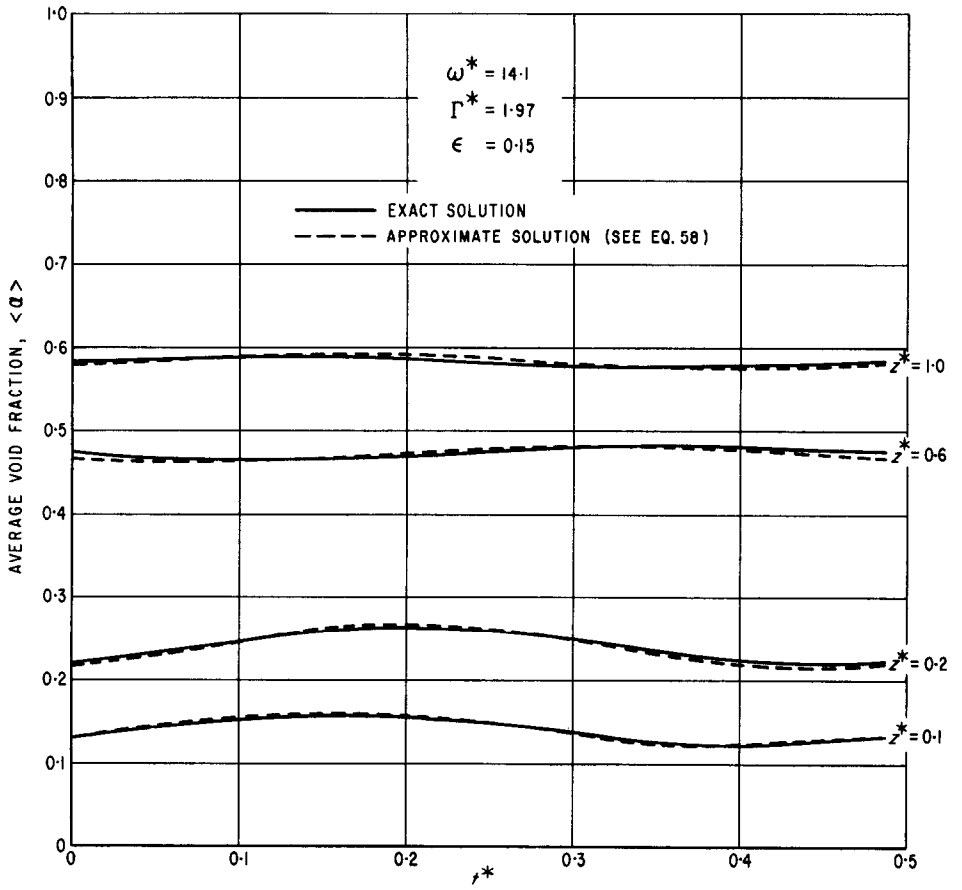

FIG. 5. Comparison of the oscillatory void fraction predicted by equation (58), with the computer solution of equation (33).

where the vapor source term is given by equation (41).

We seek now the solution of the void propagation, i.e. of equation (33) with the value of C_k^* given by equation (63). The solution is given again by equation (34) where the u_1 and u_2 are the solution of equations (37) and (38).

From equation (38), i.e. (40), we obtain the expression for the delay time t_0^* thus:

$$t_0^* = t^* + \frac{1}{\Gamma_0^*} \ln (1 - \alpha^*) = u_2 \qquad (64)$$

whereas from equations (37) and (62) we obtain:

$$u_1 = t^* - \frac{1}{\Gamma_0^*} \ln \left[1 + \Gamma_0^* z^* + \frac{v'^* \Gamma_0^*}{\omega_f^{*2} + \Gamma_0^{*2}} (\Gamma_0^* \sin \omega_f^* t^* + \omega_f^* \cos \omega^* t^*) \right]. \qquad (65)$$

The general solution is then given by equation (34), which in view of equations (64) and (65) becomes:

$$t^* + \frac{1}{\Gamma_0^*} \ln (1 - \alpha^*) = f \left\{ t^* - \frac{1}{\Gamma_0^*} \ln \left[1 + \Gamma_0^* z^* + \frac{v'^* \Gamma_0^*}{\omega_f^{*2} + \Gamma_0^{*2}} (\Gamma_0^* \sin \omega^* t^* + \omega_f^* \cos \omega_f^* t^*) \right] \right\} \qquad (66)$$

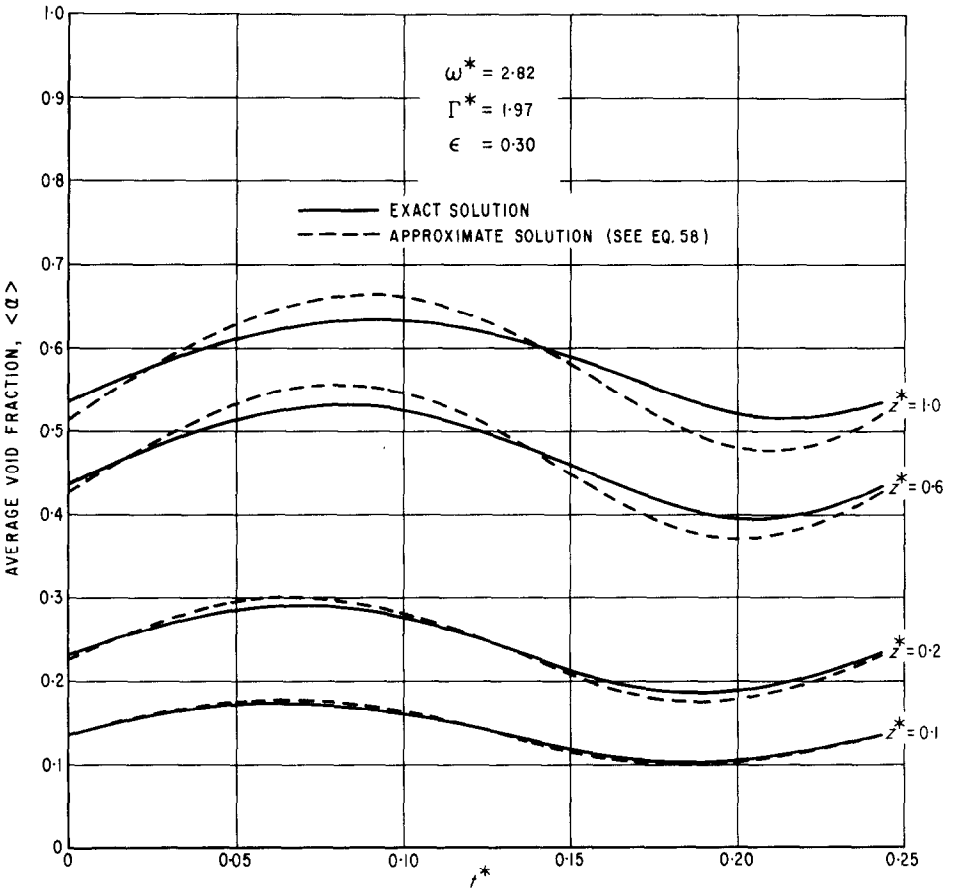

FIG. 6. Comparison of the oscillatory void fraction predicted by equation (58), with the computer solution of equation (33).

The form of the function f can be evaluated from the initial and boundary conditions. Thus for: $z^* = 0$, $\alpha^* = 0$, $t^* = t_0^*$ we find from equation (66) that:

$$f(u_1) = t^* - \frac{1}{\Gamma_0^*} \ln \left\{ \frac{1 + \Gamma_0^* z^* + (v'^* \Gamma_0^* / \omega_f^{*2} + \Gamma_0^{*2})(\Gamma_0^* \sin \omega_f^* t^* + \omega_f^* \cos \omega_f^* t^*)}{1 + (v'^* \Gamma_0^* / \omega_f^{*2} + \Gamma_0^{*2})(\Gamma_0^* \sin \omega_f^* t_0^* + \omega_f^* \cos \omega_f^* t_0^*)} \right\} \quad (67)$$

Substituting equation (67) in (66), we obtain the following expression for α^* thus:

$$1 - \alpha^* = \frac{1 + (v'^* \Gamma_0^* / \omega_f^{*2} + \Gamma_0^{*2})(\Gamma_0^* \sin \omega_f^* t_0 + \omega_f^* \cos \omega_f^* t_0^*)}{1 + \Gamma^* z^* + (v'^* \Gamma_0^* / \omega_f^{*2} + \Gamma_0^{*2})(\Gamma_0^* \sin \omega_f^* t^* + \omega_f^* \cos \omega_f^* t_0^*)} \quad (68)$$

with the delay time t_0^* given by equation (64).

In view of equation (32), it can be seen that equation (68) predicts the average volumetric concentration $\langle \alpha \rangle$, at various positions in the heated duct and at various times as function of the oscillatory inlet flow. We note that if we set the dimensionless flow amplitude equal to zero, i.e. $v'^* = 0$, in equation (68), we obtain again the solution for constant power input, i.e. equation (43).

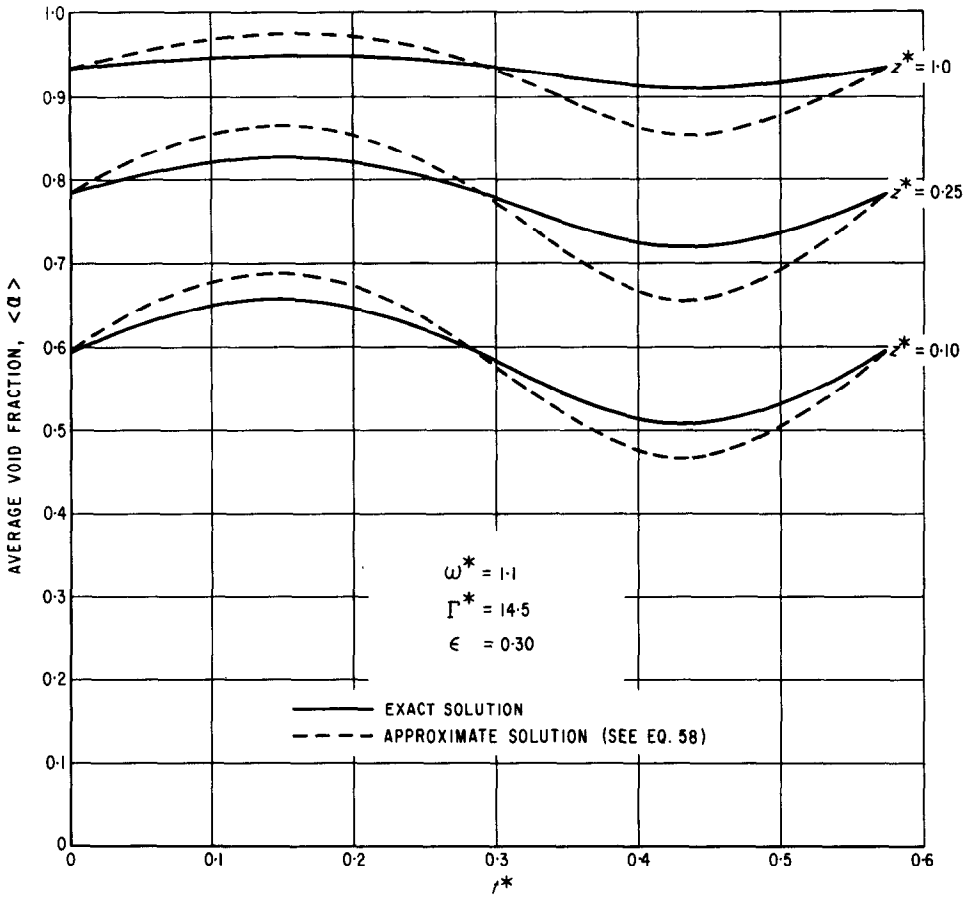

FIG. 7. Comparison of the oscillatory void fraction predicted by equation (58), with the computer solution of equation (33).

6. OSCILLATORY POWER INPUT AND OSCILLATORY INLET FLOW

In this section we shall consider the case when both power and flow oscillate. The expressions for the two oscillatory terms are given by equations (49) and (59). We shall use also the same definitions for the two frequencies given by equations (50) and (60).

The velocity of kinematic waves is given by equation (62) which, when the power also oscillates, becomes:

$$C_k^* = 1 + v'^* \sin \omega_f^* t^* + \Gamma_0^*(1 + \epsilon_p \sin \omega_p^* t^*)z^* \quad (69)$$

The dimensionless vapor source term for the case of oscillating power input is given by equation (52), i.e. by:

$$\Gamma^* = \Gamma_0^*(1 + \epsilon_p \sin \omega_p^* t^*) \quad (70)$$

where Γ_0^* is given by equation (41).

We seek now the solution of the propagation equation, i.e. of equation (33) with the values of C_k^* and of Γ^* given by equations (69) and (70) respectively. The computer solutions of this problem are shown on Figs. 10 and 11 for two cases, i.e. when the power and the flow oscillations are in phase and 180° out of phase respectively. It can be seen that the void response

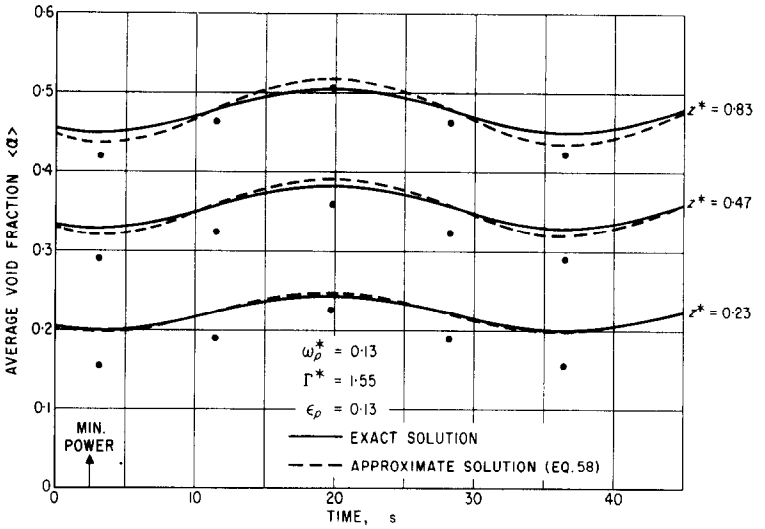

FIG. 8. Comparison of calculated and experimental void fraction for sinusoidal heat input oscillation where $\omega^* t^* = 0$ at zero time in equations (33) and (58). (Data of reference [24] for water, see Section 3.)

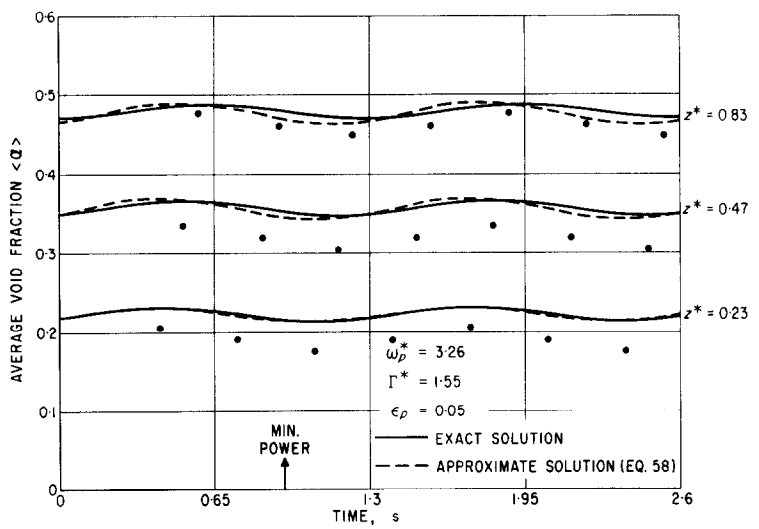

FIG. 9. Comparison of calculated and experimental void fraction for sinusoidal heat input oscillation where $\omega^* t^* = 0$ at zero time in equations (33) and (58). (Data of reference [24] for water, see Section 3.)

is most pronounced in the low void region when the two oscillations are out of phase.

It is of interest also to obtain a simple analytical solution of the problem. For this purpose, we shall introduce the same simplification which was used in Section 4, i.e. we shall neglect the effect of power oscillations on the velocity of kinematic waves. Letting, therefore, $\epsilon_p = 0$ in equation (69) and following the procedure used in Sections 4 and 5, we obtain:

$$1 - \alpha^* = \exp\left[-\Gamma_0^*(t^* - t_0^*) + \frac{\Gamma_0^* \epsilon_p}{\omega_p^*}(\cos \omega_p^* t^* - \cos \omega_p^* t_0^*)\right] \quad (71)$$

where, for given z^* and t^*, the delay time t_0^* is given by:

$$\exp\left[-\Gamma_0^*(t^* - t_0^*)\right] = \frac{1 + (v'^*\Gamma_0^*/\omega_f^{*2} + \Gamma_0^{*2})(\Gamma_0^* \sin \omega_f^* t_0 + \omega_f^* \cos \omega_f^* t_0^*)}{1 + \Gamma_0^* z^* + (v'^*\Gamma_0^*/\omega_f^{*2} + \Gamma_0^{*2})(\Gamma_0^* \sin \omega_f^* t^* + \omega_f \cos \omega_f^* t)} \quad (72)$$

In deriving equations (71) and (72), we have used $z^* = 0$, $\alpha^* = 0$, $t^* = t_0^*$, for the boundary and initial conditions.

Since α^* is related to $\langle\alpha\rangle$ by equation (32), and in view of equation (72), it can be seen that equation (71) predicts the average volumetric concentration $\langle\alpha\rangle$, as function of space and time, when both power and inlet flow oscillate. This solution is subject to the approximation of letting $\epsilon_p = 0$ in the velocity of kinematic waves given by equation (69).

Three observations can be made with respect to equations (71) and (72). First, we note that if we let $\epsilon_p = 0$ in equation (71) then this equation reduces to (68), whereas the delay time predicted by equation (72) reduces to that given by (64). Second, if we set $v'^* = 0$, then equation (71) reduces to (58), whereas (72) predicts the delay time given by (57). Finally, if we set both $\epsilon_p = 0$ and $v'^* = 0$, then equations (71) and (72) reduce to (43) and (56) respectively.

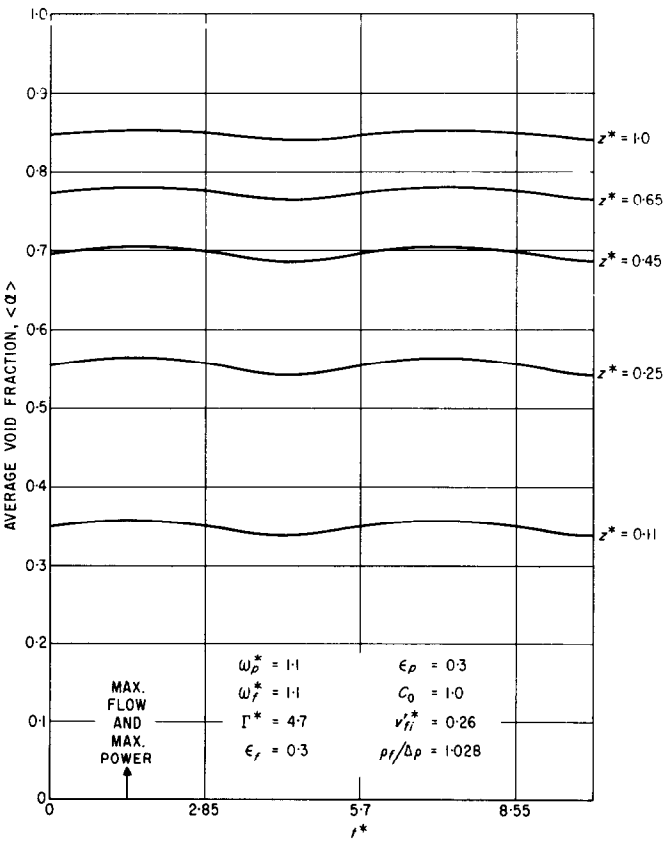

FIG. 10. Low frequency void fraction response calculated from equation (33) for in-phase inlet flow and power oscillations.

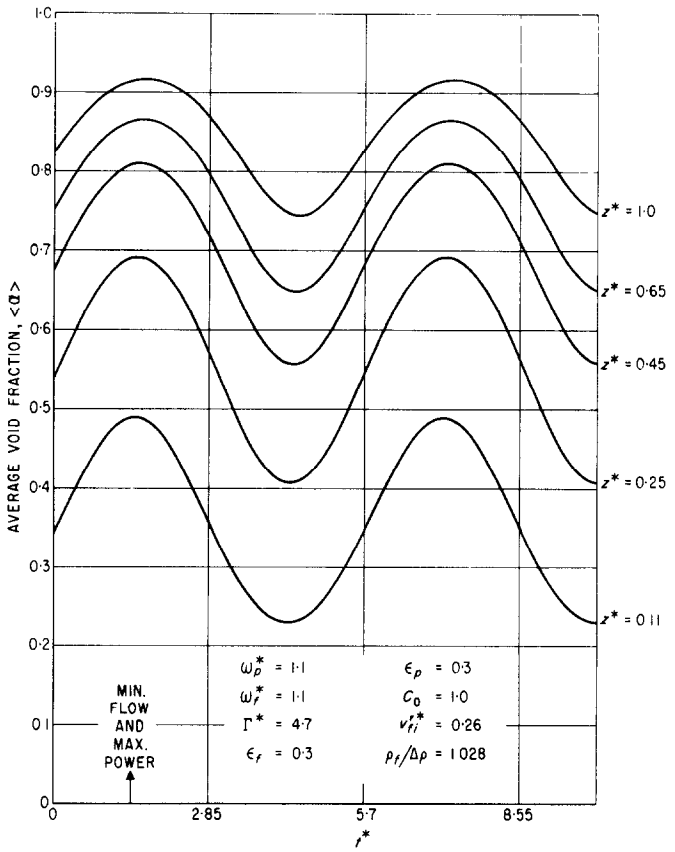

FIG. 11. Low frequency void fraction response calculated from equation (33) for 180° out of phase flow and power oscillations.

7. SUMMARY AND CONCLUSIONS

In this paper we have discussed and analysed various aspects of the void propagation equation which predicts the response of the vapor volumetric concentration to variations of (1) power, (2) inlet flow, (3) system pressure, (4) thermodynamic nonequilibrium, (5) compressibilities of the vapor and of the liquid and (6) body forces.

It is noted that since the void propagation equation predicts the void response as function of space and of time, it is particularly suitable to analyses of transients in fast, power reactors which used a liquid metal for coolant.

In this paper we have presented and discussed the solutions of the void propagation equations for the following operating conditions (1) constant power input and constant inlet flow, (2) oscillatory power input and constant flow, (3) constant power input and oscillatory flow and (4) oscillatory power and oscillatory flow. In view of the assumptions which were made in the analysis, the preceding solutions are applicable when (1) the system pressure is constant, (2) the two phases are in thermodynamic equilibrium, i.e. the inlet subcooling or superheat is negligible, (3) the compressibilities of the liquid and of the vapor are negligible, (4) the body forces acting on the mixture are constant and (5) the vapor drift velocity does not depend upon the volumetric concentration.

The analysis shows that variations of the

volumetric concentration and, therefore, the variations of the mixture density are propagated through the two-phase mixture by the velocity of kinematic waves. Expressions which predict the rate of propagation of these waves and which are appropriate to the operating conditions listed above, have been presented.

Because it takes a finite time for kinematic waves to *propagate* from one location in the system to another, the response of the volumetric concentration to various perturbations is characterized by various "delay times". The delay times, appropriate to the operating conditions enumerated above, have been also presented.

The predicted response of the vapor volumetric concentration to modulations of the power input to the fluid has been compared to experimental data for water at 400 psi in forced flow through a rectangular duct. Satisfactory agreement of predicted results with the albeit limited amount of available experimental data for water was shown. A more extensive comparison with experimental data for Refrigerant-22 in forced flow through a circular duct shows an equally satisfactory agreement.

ACKNOWLEDGEMENT

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APPENDIX A

Relation to the Stanaard Formulation in Terms of the Diffusion Equation

It has been customary in the past to analyse problems concerned with the transient behavior of the concentration in terms of the diffusion equation, in particular, in terms of the Fick's law. In what follows, we shall show, briefly, how this method can be related to the void propagation equation based on kinematic waves. A more detailed comparison is given in references [11–13].

For simplicity we consider a binary system in absence of chemical reactions; we shall neglect the effects of compressibilities. For such a system the continuity equation in terms of the volumetric concentration $\langle\alpha\rangle$ can be written [29] as:

$$\frac{\partial\alpha}{\partial t} + j\frac{\partial\alpha}{\partial z} + \frac{\partial}{\partial z}[\alpha(1-\alpha)(V_{gj} - V_{fj})] = 0 \quad \text{(A-1)}$$

It is conventional procedure in the literature [29] to express the last terms in equation (A-1) in terms of the binary diffusion coefficient $D_{\frac{1}{2}}$, thus:

$$\alpha(1-\alpha)(V_{gj} - V_{fj}) = -D_{\frac{1}{2}}\frac{\partial\alpha}{\partial z} \quad \text{(A-2)}$$

Substituting equations (A-7) and (A-4) in (A-2), we obtain:

$$\frac{\partial\alpha}{\partial t} + j\frac{\partial\alpha}{\partial z} = \frac{\partial}{\partial z}\left[D_{\frac{1}{2}}\frac{\partial\alpha}{\partial z}\right] \quad \text{(A-3)}$$

In order to relate this equation to the void propagation equation, we note that the drift velocities of the vapor and of the liquid can be expressed [17] as function of the relative velocity v_r, thus:

$$V_{gj} = (1-\alpha)v_r \quad \text{(A-4)}$$

$$V_{fj} = -\alpha v_r \quad \text{(A-5)}$$

where the relative velocity between the two phases is given by:

$$v_r = v_g - v_f \quad \text{(A-6)}$$

or in view of equations (A-4) and (A-5), it is also given by:

$$v_r = V_{gj} - V_{fj} \quad \text{(A-7)}$$

Substituting equations (A-7) and equation (A-4) in (A-2), we obtain:

$$\alpha V_{gj} = \alpha(1-\alpha)(V_{gj} - V_{fj}). \quad \text{(A-8)}$$

Whence the continuity equation, i.e. equation (A-1), can be written as:

$$\frac{\partial\alpha}{\partial t} + \left[j + \frac{\partial(\alpha V_{gj})}{\partial\alpha}\right]\frac{\partial\alpha}{\partial z} = 0 \quad \text{(A-9)}$$

which is the void propagation equation, i.e. equation (17) with the reaction frequency term Ω set equal to zero.

We note that in order to use equation (A-3), it is necessary to determine, from experiments, the value of the diffusion coefficient $D_{\frac{1}{2}}$. Experimental data on the diffusion coefficient for two-phase flow systems are almost non-existent. Another difficulty arises when the diffusion coefficient depends on the concentration; in such a case equation (A-3) becomes a nonlinear partial differential equation for which solutions are not usually available.

In view of the foregoing, it appears that, in two-phase flow systems, a formulation of the problem in terms of kinematic waves offers at least two advantages over a formulation in terms of the diffusion equation. First, expressions for

the vapor drift velocity V_{gj} in two-phase mixtures are available together with a method for determining V_{gj} [18–20]. This is not the case with the diffusion coefficient $D_{\frac{1}{2}}$. Second, it is easier to solve a first order nonlinear equation, i.e. equation (A-9), than to solve a second order nonlinear equation, i.e. equation (A-8) when $D_{\frac{1}{2}}$ is a function of α.

For further considerations and additional results, the reader is referred to references [11–13].

APPENDIX B

Relation to the Results Obtained with the Homogeneous Model

It was noted in Section 2.1, that in a two-phase flow system the velocities of the two phases are never equal, i.e. there is always a relative velocity v_r, of one phase with respect to the other. However, in a large number of papers dealing with various aspects of two-phase flow it has been assumed that this relative velocity v_r, is zero, i.e. that the two phases flow with the same velocity. It has been customary in the literature to refer to such a flow as being "homogeneous".

It is the purpose of the appendix to examine under what conditions it can be expected that the results, obtained from the "homogeneous flow" model, will be in agreement with the experimental data. This will be done by comparing the equations derived from the "homogeneous model" to those presented in Section 3.

Since it is assumed in the "homogeneous flow" model that the relative v_r, is zero, equations (A-6), (A-5) and (A-4) indicate that:

$$v_r = V_{gj} = V_{fj} = 0 \tag{B-1}$$

Thus, in the "homogeneous flow" model, the drift velocities of the vapor and of the liquid are zero.

It can be seen then from equations (7) and (8) and equation (B-1) that the velocities of the vapor and of the liquid are equal to the volumetric flux density of the mixture, thus:

$$v_g = v_f = j \tag{B-2}$$

Furthermore, it follows from equations (B-1) and (18) that in the "homogeneous flow" model:

$$C_k = j \tag{B-3}$$

It was shown elsewhere [18] that for the "homogeneous flow" model the distribution parameter C_0, given by equation (27) has a value of unity.

We use now the definition of the *flow* concentration given by

$$\beta = j_g/j \tag{B-4}$$

which can also be expressed as:

$$\beta = \frac{\dfrac{q}{\rho_g \Delta i_{fg}} \left(\dfrac{\zeta_h}{A_c}\right) z}{v_{fi} + \dfrac{\Delta\rho}{\rho_f} \dfrac{q}{\rho_g \Delta i_{fg}} \left(\dfrac{\zeta_h}{A_c}\right) z} \tag{B-5}$$

If, in accordance with the homogeneous model, we set $C_0 = 1$ and $V_{gj} = 0$ in equation (44), we obtain equation (B-5). Thus in the "homogeneous flow" model:

$$\alpha = \beta. \tag{B-6}$$

If we let $C_0 = 1$ in equation (48) we obtain the "evaporation time constant" τ derived in references [23, 24] using an approach different from that developed in Section 3, thus:

$$\frac{1}{\tau} = \frac{\Delta\rho}{\rho_f} \frac{q}{\rho_g \Delta i_{fg}} \left(\frac{\zeta_h}{A_c}\right) \tag{B-7}$$

We note that this time constant appears in most analyses (too numerous to cite them here) dealing with boiling.

Letting $C_0 = 1$ in equation (47), and recalling that $\alpha = \beta$ for "homogeneous flow" reduces equation (47) to

$$(1 - \beta)\rho_f + \beta\rho_g = \rho_f \exp\left[-\frac{t - t_0}{\tau}\right] \tag{B-8}$$

where τ is given by equation (B-7). This equation gives the density of the mixture for "homogeneous flow". It was derived also in [25–28] using a different method.

It can be concluded from the foregoing that if we let $C_0 = 1$ and $V_{gj} = 0$ in the equations which were derived in Section 3, then the results of that section reduce to those obtained previously for the "homogeneous flow" model. However, since the drift velocity of the vapor is never equal to zero it can be concluded that the results predicted by the "homogeneous flow" model will be in satisfactory agreement with the data *only* if $v_{fi} \gg V_{gj}$. This will be true for high mass flow rates. The second condition, i.e. that $C_0 = 1$ is approximately satisfied at high mass flow rates as well as in the fog flow regime.

Conversely, it can be expected that the greatest discrepancy between the results predicted by the "homogeneous flow" model and the experimental data will occur at low mass flow rates when the inlet liquid velocity and the vapor drift velocity are of the same order of magnitude.

We note further that, in contrast to the results presented in this paper, the "homogeneous flow" model does not predict the effect of the two-phase flow regime on the transient response of the volumetric concentration. This statement is based on the fact that for the "homogeneous flow" model $C_0 = 1$ and $V_{gj} = 0$ whereas a change of flow regimes implies changes of both the distribution parameter C_0, and of the vapor drift velocity V_{gj}.

Résumé—Différents aspects et différentes caractéristiques de l'équation de propagation des vides sont discutés. Cette équation permet d'obtenie la réponse transitoire de la concentration volumique aux perturbations de (1) la puissance d'entrée, (2) du flux d'entrée, (3) de la pression du système, (4) du déséquilibre thermodynamique, (5) des compressibilités de la vapeur et du liquide et (6) des forces volumiques agissant sur le mélange diphasique. Cette réponse transitoire est alors à la fois fonction des coordonées spatiales et fonction du temps.

On a obtenu les solutions de l'équation de propagation des vides pour les conditions opératoires suivantes: (1) puissance et flux d'entrée constants, (2) puissance d'entrée oscillatoire, (3) flux d'entrée oscillatoire et (4) puissance et écoulement oscillatoires.

On montre que les perturbations de la densité du mélange se propagent à travers le mélange diphasique à la *vitesse des ondes cinématiques*. Les expressions qui prédisent la vitesse de propagation de ces ondes et qui correspondent aux conditions opératoires citées ci-dessus sont données.

La vitesse de propagation finie des ondes cinématiques introduit un "retard" qui caractérise la réponse de la concentration volumique aux différentes perturbations. Les "retards" propres aux conditions opératoires énumérées ci-dessus sont également présentés. Les résultats prédits sont comparés aux données expérimentales disponibles avec un accord satisfaisant.

Zusammenfassung—Verschiedene Gesichtspunkte und Charakteristika der Ausbreitungsgleichung des Dampfes werden diskutiert. Diese Gleichung bestimmt das Übergangsverhalten der volumetrischen Konzentration gegen Strörugen (1) der Energiezufuhr, (2) der Einlass-Strömung, (3) des Systemdruckes, (4) des thermodynamischen Ungleichgewichts, (5) der Kompressibilitäten von Dampf und Flüssigkeit und (6) der auf das Zweiphasengemisch wirkenden Massenkräfte. Dieses Übergangsverhalten wird für Funktionen des Ortes und der Zeit bestimmt.

Lösungen der Ausbreitungsgleichung sind für folgende Arbeitsbedingungen hergeleitet: (1) konstante Energie und Einlass-Strömung, (2) oszillierende Energiezufuhr, (3) oszillierende Einlass-Strömung und (4) oszillierende Energie und oszillierende Strömung.

Es wird gezeigt, dass Störungen der Gemischdichte im Zweiphasengemisch entsprechend der Greschwindigkeit der kinematischen Wellen ausgebreitet werden. Ausdrücke, die den oben angeführten Arbeitsbedingungen angepasst sind und zur Berechnung der Ausbreitungsrate dieser Wellen dienen, werden angegeben.

Die endliche Ausbreitungsrate der kinematischen Wellen führt auf eine "Verzögerungszeit", die das Ansprechen der volumetrischen Konzentration auf verschiedene Störungen charakterisiert. Die zu den Arbeitsbedingungen gehörigen "Verzögerungszeiten" sind ebenfalls erwähnt.

Die ermittelten Ergebnisse werden mit verfügbaren Versuchsdaten verglichen. Die Übereinstimmung ist zufriedenstellend.

Аннотация—Рассматриваются различные типы и характеристики уравнения распространения пузырьков. Это уравнение определяет неустановившуюся реакцию объёмной концентрации на возмущения (1) подводимой мощности, (2) входящего потока, (3) давления системы, (4) термодинамического неравновесия, (5) сжимаемости пара и жидкости и (6) массовых сил, действующих на двухфазную смесь. Эта неустановившаяся реакция определена как функция пространства и времени.

Решения уравнения распространения пузырьков выведены для следующих рабочих условий: (1) постоянные мощности и входящий поток, (2) осциллирующая подводимая мощность, (3) осциллирующий входящий поток, (4) осциллируюая мощность и осциллирующий поток.

Показано, что возмущения плотности смеси распространяются через двухфазную смесь со скоростью кинематических волн. Приводятся выражения, по которым рассчитывается скорость распространения этих волн и которые соответствуют перечисленным выше рабочим условиям.

Конечная скорость распространения кинематических волн вводит «время запаздывания», которое характеризует реакцию объёмной концентрации на различные возмущения. Приводится «время запаздыкания», соответствующее рабочим условиям, перечисленным выше.

Сравнение полученных результатов с имеющимися экспериментальными данными показывает удовлетворительное соответствие между ними.